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Effect of Common Process Noise on Two-Sensor Track Fusion

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In a multisensor environment for surveillance systems, each sensor tracks multiple targets. It is assumed that sensors are equipped with optimal Kalman filters for target tracking. These tracks are correlated because the common process noise resulting from target maneuver enters the estimate of the state vector of the target being tracked. To obtain better quality tracks, the tracks from each sensor are associated using the nearest neighbor criterion for track matching and then kinematic track fusion is performed using the matched tracks. For this purpose, the cross-correlation matrix between tracks is introduced in the test statistic to test the hypothesis that the two tracks originated from the same target. It is shown that the probability distribution of correct track association is increased if the cross-covariance matrix introduced in the test statistic is positive. Necessary and sufficient conditions for the existence, uniqueness, and positivity of the cross-covariance matrix are derived. In addition, an expression for the steady-state cross-covariance matrix is obtained, which is shown to be a function of the parameters of the two filters associated with the candidate tracks being fused. It is shown that for two identical sensors, if the cross-covariance matrix is to be positive definite, certain restrictions on steady-state performance of the individual Kalman filters must be placed. Other measures of performance on the effect of cross correlation on kinematic track fusion are also discussed.

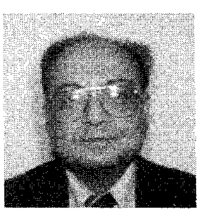
Nomenclature

CS = closeness score for independent tracks
CS^* = closeness score for dependent tracks
$E[\,]$ = statistical expectation
G = coefficient matrix of the driving noise
H = observation matrix
H_0 = null hypothesis
H_1 = nonnull hypothesis
K^m = Kalman gain matrix for mth filter, $m = 1, 2$
P_{CA} = probability distribution of correct association for independent tracks
P^*_{CA} = probability distribution of correct association for dependent tracks
$P^C(t_k/t_k)$ = cross-covariance matrix at time t_k
$P^F(t_k/t_k)$ = fused track covariance at time t_k
$P^m(t_k/t_k)$ = filter covariance matrix for mth filter, $m = 1, 2$
$P^m(t_{k+1}/t_k)$ = one-step predictor covariance matrix for mth filter, $m = 1, 2$
$Pr(\,)$ = probability distribution of ( )
Q = target maneuver noise variance
$R(\,)$ = range of an operator ( )
R_n = Euclidean norm of a vector of dimension n
T = sampling time interval
V^m = measurement noise of mth sensor, $m = 1, 2$
W = target maneuver noise
X = target state vector
$\hat{X}^m(\,)$ = conditional mean estimate of target state vector from mth sensor
α = type I error for independent tracks
α^* = type I error for dependent tracks
$\lambda_{\max}$ = maximum eigenvalue of a matrix
$\lambda_{\min}$ = minimum eigenvalue of a matrix
λ_n = test threshold for chi-square distribution with n degrees of freedom
σ_x^m = standard deviation of position observation noise of mth sensor, $m = 1, 2$
ϕ = state transition matrix for target state vector
$\|(\,)\|$ = norm of a vector or matrix ( )
$\otimes$ = Kronecker product

Introduction

IN recent years, there has been increased recognition by the command, control, communication and intelligence community of the need to perform track-to-track fusion. This interest has been heightened by the availability of sophisticated sensors, which exploit different characteristics of the optical, infrared, and electromagnetic spectrums for tracking multiple targets. After acquisition of target data and formation of tracks, there is a need to associate tracks obtained from these sensors, which originated from the same target.

This problem was addressed by several authors[1–3] who assumed that the estimation errors of tracks from different sensors that are tracking the same target are uncorrelated. It was shown in Ref. 4 that this assumption is incorrect because the process noise associated



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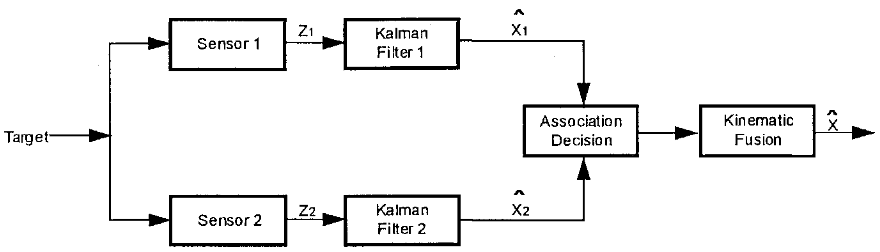

Fig. 1 Track-to-track fusion.

with target maneuvering enters into the filter dynamics, used to create tracks at each sensor site. Hence, the target tracks are correlated through the process noise even though the sensor measurement errors are uncorrelated. The author[4] proposed a superior statistical test for association by exploiting the cross correlation between the tracks. In a subsequent paper,[5] an algorithm for track fusion by combining the track estimates obtained from different sensors was also proposed.

An alternative approach to kinematic state vector fusion was proposed in Ref. 6, which combines the measurements from different sensors to obtain the best estimate of the target state. Since the measurement fusion approach uses all of the available information, it has been shown[7,8] that this approach is optimal and results in reduction of the state vector error covariance as compared to the state vector fusion approach.[5] However, this advantage is diminished[9] if the two sensors have widely varying measurement noise variances, which essentially means that if the quality of one sensor exceeds that of the other by a wide margin, then it is not worthwhile to make use of the low-quality sensor's kinematic state estimate for the purpose of track fusion. However, the low-quality sensor might still contain useful information for identifying the target type.

In this paper, it is assumed that two sensors are employed to track multiple targets. The sensors detect the targets and employ optimal Kalman filters to create two sets of track files. Since the input process noise resulting from target maneuver is used in the Kalman filters, the two sets of track files are correlated. These two sets of tracks are then associated using the nearest neighbor criterion by introducing the cross-covariance matrix[4] in the test statistic for track matching. This test statistic is used to test the hypothesis that the two tracks originated from the same target. It is shown that the probability distribution of correct track-to-track association is increased if the cross-covariance matrix introduced in the test statistic is positive. In addition, a closed-form steady-state solution of the cross covariance is obtained by considering optimal fusion of track state estimates when only the target position is observed by two identical sensors. Results are validated by simulating a two-state (position and velocity) variable target kinematic model.

Mathematical Model

For the sake of simplicity, it is assumed that two identical sensors are tracking a target. After acquisition, the target is tracked with a Kalman filter associated with the sensor. It is assumed that the kinematic model of a tracked target is described by

$$X(t_{k+1}) = \phi X(t_k) + GW(t_k) \tag{1}$$

where the target state vector $X(t_k)$ is modeled by a two-state (position and velocity) variable and

$$\phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \tag{2}$$

The input noise $W(t_k)$ is described by a white Gaussian noise, where

$$E[W(t_k)] = 0 \qquad \mathrm{Var}[W(t_k)] = Q = \sigma_a^2 \tag{3}$$

The measurement at each sensor is given by

$$Z^m(t_k) = HX(t_k) + V^m(t_k) \qquad m = 1, 2 \tag{4}$$

where the measurement noise is assumed to be white Gaussian and

$$H = [1 \quad 0]$$
$$E[V^m(t_k)] = 0 \qquad \mathrm{Var}\big[V^m(t_k)\big] = \sigma_x^{m2} \qquad m = 1, 2 \tag{5}$$

The fusion process is described in Fig. 1. Estimates of the target state vectors are obtained by each sensor employing the following optimal linear Kalman filters:

$$\hat{X}^m(t_{k+1}/t_{k+1}) = \phi\hat{X}^m(t_k/t_k) + K^m(t_{k+1})\big[Z^m(t_{k+1}) - H\phi\hat{X}^m(t_k/t_k)\big] \tag{6}$$

$$K^m(t_{k+1}) = P^m(t_{k+1}/t_k)H^T\big[HP^m(t_{k+1}/t_k)H^T + \sigma_x^{m2}\big]^{-1} \tag{7}$$

$$P^m(t_{k+1}/t_k) = \phi P^m(t_k/t_k)\phi^T + GQG^T \tag{8}$$

$$P^m(t_{k+1}/t_{k+1}) = \big[I - K^m(t_{k+1})H\big]P^m(t_{k+1}/t_k) \qquad m = 1, 2 \tag{9}$$

This model assumes that both the filters use the same state transition matrix ϕ, which matches target dynamics, and the same observation matrix H. If the filter models are different, then the analysis becomes complicated.

Association Algorithm

For two-sensor track-to-track fusion, the association algorithm involves testing the hypothesis at every t_k that the two tracks $\hat{X}^1$ and $\hat{X}^2$ originated from the same target.[10] Hence, the null hypothesis can be stated as H_0: $\hat{X}^1 - \hat{X}^2 = 0$ vs H_1: $\hat{X}^1 - \hat{X}^2 \neq 0$. If the tracks are independent,[1] then under hypothesis H_0, $\mathrm{cov}(\hat{X}^1 - \hat{X}^2) = P^1 + P^2$ and assuming Gaussian probability distribution of the track estimates, the test of hypothesis can be restated as H_0: $(\hat{X}^1 - \hat{X}^2)^T(P^1 + P^2)^{-1}(\hat{X}^1 - \hat{X}^2) \leq \lambda_n$, where λ_n defines the gate width of the association region. Since the test statistic $(\hat{X}^1 - \hat{X}^2)^T(P^1 + P^2)^{-1}(\hat{X}^1 - \hat{X}^2)$ is chi-square with n degrees of freedom, the test threshold λ_n can be chosen to guarantee the goodness of fit for any level of significance. Hence, the threshold is such that

$$Pr\big\{(\hat{X}^1 - \hat{X}^2)^T(P^1 + P^2)^{-1}(\hat{X}^1 - \hat{X}^2) > \lambda_n \big| H_0\big\} = \alpha$$

where α is, say, 0.1. This test statistic was modified by several authors[3,11] by constructing the closeness score $CS(t_M)$ defined by the following test statistic:

$$CS(t_M) = \sum_{k=1}^{M} [\hat{X}^1(t_k/t_k) - \hat{X}^2(t_k/t_k)]^T \times [P^1(t_k/t_k) + P^2(t_k/t_k)]^{-1}[\hat{X}^1(t_k/t_k) - \hat{X}^2(t_k/t_k)] \tag{10}$$

which is a cumulative sum over M track match points. The probability distribution of correct association of two tracks $\hat{X}^1$ and $\hat{X}^2$, denoted by P_{CA}, is then obtained by assuming independence[1] of the error between the two track estimates and by testing $CS(t_M)$ against the threshold λ_{nM} corresponding to the chi-square distribution with nM degrees of freedom:

$$P_{CA} = Pr\{CS(t_M) \leq \lambda_{nM} | H_0\} = 1 - \alpha \tag{11}$$

As the number of track matching time points increase, the closeness score increases, and it can be shown[1] that the probability distribution of correct association converges to 1 almost everywhere. To incorporate the dependence between the track estimates[4] because of the common process noise, the closeness score, $CS(t_M)$ defined in Eq. (10) is modified as

$$CS^*(t_M) = \sum_{k=1}^{M} [\hat{X}^1(t_k/t_k) - \hat{X}^2(t_k/t_k)]^T$$
$$\times P^E(t_k/t_k)^{-1}[\hat{X}^1(t_k/t_k) - \hat{X}^2(t_k/t_k)] \tag{12}$$

where

$$P^E(t_k/t_k) = P^1(t_k/t_k) + P^2(t_k/t_k) - P^C(t_k/t_k) - P^C(t_k/t_k)^T \tag{13}$$

and

$$P^C(t_k/t_k) = \left[I - K^1(t_k)H\right]\phi P^C(t_{k-1}/t_{k-1})\phi^T\left[I - K^2(t_k)H\right]^T$$
$$+ \left[I - K^1(t_k)H\right]GQG^T\left[I - K^2(t_k)H\right]^T \tag{14}$$

with

$$P^C(0/0) = 0$$

Once the decision to associate two tracks has been made using $CS^*(t_M)$ as the test statistic, the estimate of the kinematically fused track and its covariance now become[5]

$$\hat{X}^F(t_k/t_k) = \hat{X}^1(t_k/t_k) + \left[P^1(t_k/t_k) - P^C(t_k/t_k)\right]$$
$$\times P^{E^{-1}}(t_k/t_k)\left[\hat{X}^2(t_k/t_k) - \hat{X}^1(t_k/t_k)\right] \tag{15}$$

$$P^F(t_k/t_k) = P^1(t_k/t_k) - \left[P^1(t_k/t_k) - P^C(t_k/t_k)\right]$$
$$\times P^{E^{-1}}(t_k/t_k)\left[P^1(t_k/t_k) - P^C(t_k/t_k)\right]^T \tag{16}$$

Incorporating the cross covariance in the modified closeness score, the probability distribution of correct association denoted by P^*_{CA} becomes

$$P^*_{CA} = Pr\left[CS^*(t_M) \le \lambda^*_{nM}\middle|H_0\right] = 1 - \alpha^* \tag{17}$$

The following theorem shows that incorporation of cross covariance in the closeness score results in improved performance of the track matching algorithm for track association:

Theorem 1: Let the matrices P^1, P^2 be positive definite and the asymmetric matrix P^C be positive. Then

$$\alpha^* < \alpha \tag{18}$$

Proof: To prove this theorem, it is useful to recognize that both $CS(t_M)$ and $CS^*(t_M)$ can be written as quadratic forms and are positive if $\hat{X}^1 - \hat{X}^2 \neq 0$ and P^C is positive. Hence, the closeness score is monotonically increasing, which leads to Eq. (18) if and only if $CS^*(t_M) > CS(t_M)$ for all t_M.

Let the norm of a vector $X = (X_1, X_2, \ldots, X_n)$ be defined as

$$\|X\| = \sum_{i=1}^{n} X_i^2$$

and let the eigenvalues of an $n \times n$ positive definite matrix A be such that

$$0 < \lambda_1 = \lambda_{\min}, \lambda_2, \ldots, \lambda_n = \lambda_{\max} < \infty$$

and let C be a compact set in the Euclidean space R_n. Also, let the norm of a matrix A be defined as

$$\|A\| = \operatorname*{Sup}_{X \neq 0} \frac{X^T A X}{X^T X}$$

Now, for each t_k, since P^C is a cross-covariance matrix, $\|P^m - P^C\| > 0$, $m = 1, 2$, and it is possible to write

$$[\hat{X}^1 - \hat{X}^2]^T P^{E^{-1}}[\hat{X}^1 - \hat{X}^2]$$
$$\ge \lambda_{\min}\left(P^{E^{-1}}\right)\|\hat{X}^1 - \hat{X}^2\|$$
$$= \frac{1}{\left\|P^1 + P^2 - P^C - P^{C^T}\right\|}\|\hat{X}^1 - \hat{X}^2\|$$
$$\ge \frac{1}{\|P^1 + P^2\|}\|\hat{X}^1 - \hat{X}^2\| \qquad \text{iff} \quad \|P^C\| \ge 0 \tag{19}$$

Since Eq. (19) is valid for all track matching times t_k, summing M such positive terms yields

$$CS^*(t_M) \ge CS(t_M) \qquad \forall \quad t_M \tag{20}$$

Now, since both $CS^*(t_M)$ and $CS(t_M)$ are monotonically increasing and $CS^*(t_M)$ dominates $CS(t_M)$ if and only if $\|P^C\| \ge 0$, it is possible to write

$$1 - \alpha = P_{CA} = P_r[CS(t_M) \le \lambda_{nM}|H_0]$$
$$\le P_r\left[CS^*(t_M) \le \lambda^*_{nM}\middle|H_0\right] = 1 - \alpha^* \tag{21}$$

which implies that $\alpha^* < \alpha$ if and only if the cross-covariance matrix P^C is positive. Note that when the two sensors are identical, so that

$$P^1 = P^2 = P \qquad \text{and} \qquad P^C = P^{C^T} \tag{22}$$

then

$$P^{E^{-1}} = \tfrac{1}{2}(P - P^C)^{-1} \tag{23}$$

Since P^C is a cross-covariance matrix, $\|P - P^C\| > 0$. But $CS^*(t_M) \ge CS(t_M)$, and $P^*_{CA} \ge P_{CA}$ if and only if $\|P^C\| \ge 0$. Thus, benefit from using the modified test statistic can only be achieved if the cross-covariance matrix is positive definite. Furthermore, stronger cross covariance results in higher probability of track-to-track association. For two identical sensors, conditions for the existence, uniqueness, and positive definiteness of the steady-state solution of the cross-covariance matrix P^C given in Eq. (14) is considered in the next section.

Steady-State Analysis of Cross Covariance

In this section, it is assumed that two identical sensors are tracking the same target and the measurement noise statistics of the two sensors are identical. Hence,

$$\left(\sigma_x^1\right)^2 = \left(\sigma_x^2\right)^2 = (\sigma_x)^2 \tag{24}$$

In this case, the Kalman filter gain and covariance of the state estimates are the same, so that each of the sensor tracks can be described as

$$\hat{X}(t_{k+1}/t_{k+1}) = \phi\hat{X}(t_k/t_k) + K(t_{k+1})[Z(t_{k+1}) - H\phi\hat{X}(t_k/t_k)] \tag{25}$$

where

$$K(t_k) = \begin{bmatrix} K_1(t_k) \\ K_2(t_k) \end{bmatrix} \tag{26}$$

Since the covariance matrices are also the same for the two trackers, the superscript m is removed, henceforth, from Eqs. (6–9). The recursive relationship for the cross covariance at steady state can be written from Eq. (14) as

$$P^C = [I - KH]\phi P^C \phi^T [I - KH]^T$$
$$+ [I - KH]GQG^T[I - KH]^T \tag{27}$$

or

$$P^C = FP^CF^T + Q^* \tag{28}$$

where

$$F = [I - KH]\phi$$

or

$$F = \begin{bmatrix} 1-K_1 & T(1-K_1) \\ -K_2 & 1-TK_2 \end{bmatrix} \tag{29}$$

and

$$Q^* = [I - KH]GQG^T[I-KH]^T$$
$$= \sigma_a^2 T^2 \begin{bmatrix} \frac{T^2}{4}(1-K_1)^2 & \frac{T}{2}(1-K_1)\left(1-\frac{T}{2}K_2\right) \\ \frac{T}{2}\left(1-\frac{T}{2}K_2\right)(1-K_1) & \left(1-\frac{T}{2}K_2\right)^2 \end{bmatrix} \tag{30}$$

which are obtained by substituting for ϕ, G, Q, and H given in Eqs. (2–5).

Equation (28) is the familiar discrete Lyapunov equation whose solution, obtained in the Appendix, is given as

$$\begin{bmatrix} P^C(1,1) \\ P^C(1,2) \\ P^C(2,1) \\ P^C(2,2) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} [1+F^2(2,2)](1+\det F) - 2F(2,2)\mathrm{tr}F & -F(1,2)[(1+\det F)F(2,2) - \mathrm{tr}F] & -F(1,2)[F(2,2)(1+\det F) - \mathrm{tr}F] & F^2(1,2)(1+\det F) \\ -F(2,1)[F(2,2)(1+\det F) - \mathrm{tr}F] & [1+F(1,1)F(2,2)](1+\det F) - (\mathrm{tr}F)^2 & F(1,2)F(2,1)(1+\det F) & -F(1,2)[F(1,1)(1+\det F) - \mathrm{tr}F] \\ -F(2,1)[F(2,2)(1+\det F) - \mathrm{tr}F] & F(1,2)F(2,1)(1+\det F) & [1+F(1,1)F(2,2)](1+\det F) - (\mathrm{tr}F)^2 & [1+F(1,1)F(2,2)](1+\det F) - (\mathrm{tr}F)^2 \\ F^2(2,1)(1+\det F) & -F(2,1)[F(1,1)(1+\det F) - \mathrm{tr}F] & -F(2,1)[F(1,1)(1+\det F) - \mathrm{tr}F] & [1+F^2(1,1)](1+\det F) - 2F(1,1)\mathrm{tr}F \end{bmatrix} \begin{bmatrix} Q^*(1,1) \\ Q^*(1,2) \\ Q^*(2,1) \\ Q^*(2,2) \end{bmatrix} \tag{31}$$

where

$$\Delta = (1 - \det F)\det(F - I)\det(F + I) \tag{32}$$

In Eq. (31), elements of a matrix A are represented as $A(i, j)$, $i, j = 1, 2$, and det A denotes the determinant of the matrix A and tr A denotes its trace. It is also stated in the Appendix that for the steady-state cross-covariance matrix to be positive definite, the determinant Δ given in Eq. (32) must be positive. For this condition to be satisfied, the following constraints must be satisfied.

Constraint 1:

$$1 - \det F > 0 \quad \text{or} \quad 1 - \begin{vmatrix} 1-K_1 & T(1-K_1) \\ -K_2 & 1-TK_2 \end{vmatrix} > 0 \quad \text{or} \quad K_1 > 0 \tag{33}$$

Constraint 2:

$$\det(F - I) > 0 \quad \text{or} \quad \begin{vmatrix} -K_1 & T(1-K_1) \\ -K_2 & -TK_2 \end{vmatrix} > 0 \quad \text{or} \quad K_2 > 0 \tag{34}$$

Constraint 3:

$$\det(F + I) > 0 \quad \text{or} \quad \begin{vmatrix} 2-K_1 & T(1-K_1) \\ -K_2 & 2-TK_2 \end{vmatrix} > 0$$
$$\text{or} \quad 4 - 2K_1 - TK_2 > 0 \tag{35}$$

The first two conditions stated in Eqs. (33) and (34) have been established for the optimal steady-state Kalman filter using this particular two-state system and observation model by various authors[12–14] and is well known. Existence of the third condition given in Eq. (35) was established by other researchers[15,16] while investigating the stability of the steady-state behavior of α–β filters. Hence, this constraint appears as a necessary condition for the cross-covariance matrix to be positive definite, which is also necessary for establishing improved track-to-track association using the nearest neighbor algorithm. Substituting the elements of F and Q^* matrices from Eqs. (29) and (30) into Eqs. (31) and (32) yields

$$P^C = \frac{\sigma_a^2 T^2}{2K_1 T K_2} \times \begin{bmatrix} T^2(1-K_1)^2 & T(1-K_1)\left(K_1 - \frac{TK_2}{2}\right) \\ T(1-K_1)\left(K_1 - \frac{TK_2}{2}\right) & K_1\left(K_1 - \frac{TK_2}{2}\right) + TK_2(1-K_1) \end{bmatrix} \tag{36}$$

The steady-state solution of P^C given in closed form by Eq. (36) is of considerable interest in the data fusion community. It spells out the effect of the position and velocity gains of the individual sensor Kalman filters used for tracking the target on the cross covariance between the tracks resulting from common process noise. Also, knowledge of the exact nature of cross coupling between position and velocity states could be utilized in Eqs. (12) and (13) to derive suboptimal closeness score for track matching. For example, the off-diagonal terms of Eq. (36) can be neglected and/or only the first-order terms involving the Kalman filter gains be retained.

Numerical Results

Two-sensor, state-vector fusion steady-state performance incorporating the cross covariance between the two sensors was analyzed by simulating two identical Kalman filters with steady-state gains of[14]

$$K_1 = P(1,1) \tag{37}$$

$$K_2 = \frac{P(1,2)}{T} \tag{38}$$

and filter covariances of

$$P(1,1) = [0.5(r + 4\sqrt{r})]\left[\left\{1 + \frac{4}{r + 4\sqrt{r}}\right\}^{\frac{1}{2}} - 1\right] \tag{39}$$

$$P(1,2) = \frac{P(1,1)^2 + r[1 - P(1,1)]}{2 - P(1,1)} \tag{40}$$

$$P(2,2) = \frac{P(1,1)P(1,2)}{1 - P(1,1)} - 2r \tag{41}$$

where

$$r = \left[\frac{\sigma_a(T^2/2)}{\sigma_x}\right]^2 \tag{42}$$

Since the two sensors are identical, Eq. (16) reduces to

$$P^F = \tfrac{1}{2}(P + P^C) \tag{43}$$

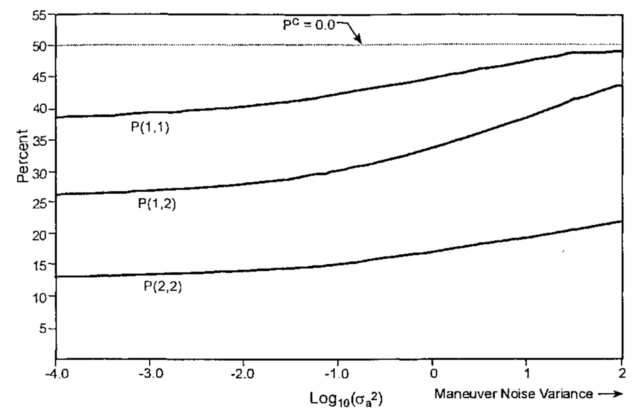

Fig. 2 Performance improvement of track-to-track fusion because of incorporation of cross-covariance, σ_x = 1.0 ft (rms).

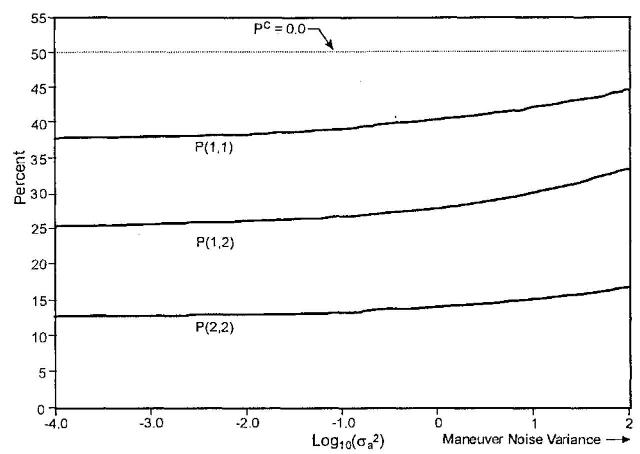

Fig. 3 Performance improvement of track-to-track fusion because of incorporation of cross covariance, σ_x = 10.0 ft (rms).

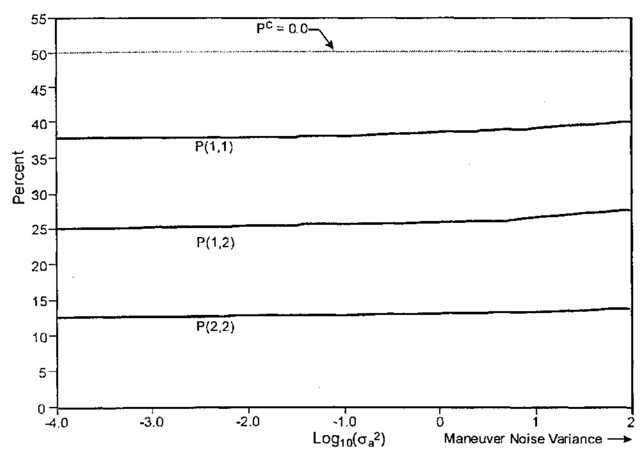

Fig. 4 Performance improvement of track-to-track fusion because of incorporation of cross covariance, σ_x = 100.0 ft (rms).

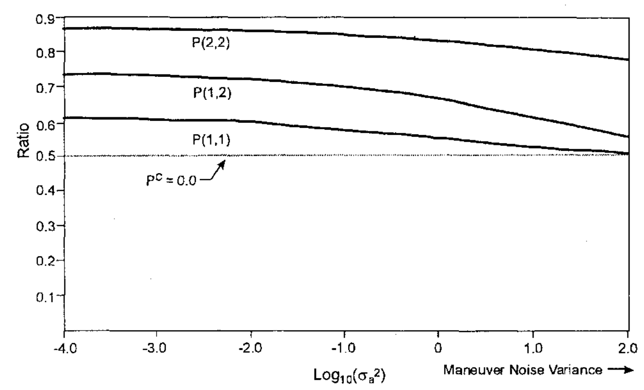

Fig. 5 Ratio of components of fused covariance matrix to single sensor track covariance, σ_x = 1.0 ft (rms).

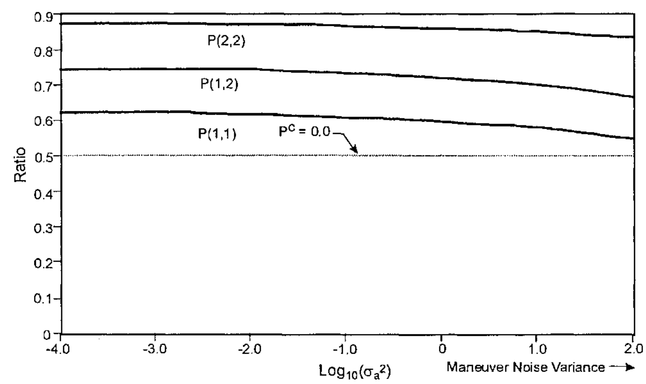

Fig. 6 Ratio of components of fused covariance matrix to single sensor track covariance, σ_x = 10.0 ft (rms).

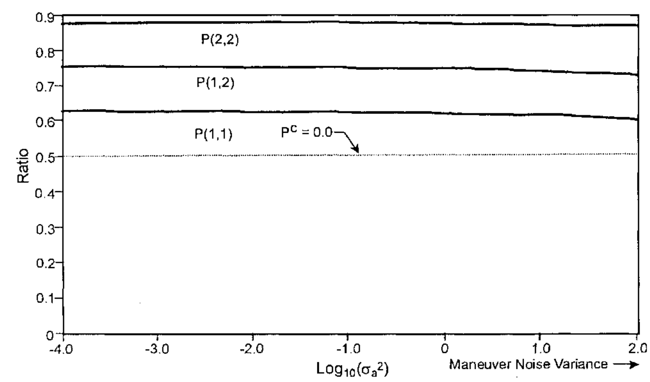

Fig. 7 Ratio of components of fused covariance matrix to single-sensor track covariance, σ_x = 100.0 ft (rms).

and

$$P - P^F = \tfrac{1}{2}(P - P^C) \tag{44}$$

If the cross-covariance matrix P^C is zero, then Eq. (44) shows that $P^F = P/2$. Thus, two-sensor state vector fusion leads to a 50% reduction of individual track filter covariance. To quantify the improvement resulting from fusion when P^C is not zero, elements of the matrix $P - P^F$ are converted to a percentage of the corresponding element of the matrix P and plotted in Figs. 2–4 for a wide range of values of σ_a^2. These three figures correspond to three different values of sensor measurement noise, $\sigma_x = 1$, 10, and 100 ft (rms). The results show that the percent improvement resulting from fusion by incorporating cross covariance increases considerably as the maneuver noise level σ_a (ft/s^2) is increased. However, this improvement is less pronounced if the sensors are of poor quality, characterized by increasing the measurement noise level σ_x. These results are also plotted in Figs. 5–7 as ratios of the elements of fused covariance matrix to that of a single-sensor state estimate covariance, which are in agreement with the results in Refs. 5 and 8.

Conclusions

The effects of cross correlation because of maneuver noise on track-to-track association and the kinematic state vector fusion have been quantified when a target is tracked by two identical sensors. It has been shown that the performance of the nearest neighbor track-to-track association algorithm can be improved if and only if the cross covariance between the candidate tracks for association is positive definite. An efficient algorithm for derivation of steady-state cross covariance in closed form has been presented and conditions for positive definiteness of this matrix have also been derived. Numerical simulation indicates that incorporation of the cross covariance in the fusion algorithm results in superior performance as compared to track fusion with zero cross covariance.

Appendix A: Steady-State Solution of Discrete Lyapunov Equation

It is well known that application of Lyapunov's second method[17] to investigate the stability of an nth-order discrete-time constant linear system described by

$$X(t_{k+1}) = FX(t_k) \tag{A1}$$

requires solution of the algebraic equation of the type

$$P = FPF^T + Q^* \tag{A2}$$

It has been shown that the linear system (A1) is asymptotically stable if and only if the solution for the symmetric matrix P of Eq. (A2) is positive definite, given an arbitrary positive definite symmetric matrix Q^*. In this case, the cost function $X^T(t_k)PX(t_k)$ is a Lyapunov function for Eq. (A1), and Eq. (A2) is often referred to as the discrete Lyapunov equation. Hence, to investigate whether the system (A1) is stable, it is necessary to know whether a solution of Eq. (A2) exists and, if so, whether it is unique and under what conditions such a solution is positive definite.

Conditions for Existence, Uniqueness, and Positive Definiteness

Since the matrices P and Q^* are assumed to be symmetric, Eq. (A2) can be written as a system of $n(n+1)/2$ linear algebraic equations for the $n(n+1)/2$ unknown elements of P:

$$\bar{F}p = q^* \tag{A3}$$

where

$$\bar{F} = I - F \otimes F^T$$

The symbol $\otimes$ denotes the Kronecker product [the Kronecker product of two matrices $A(p \times q)$ and $B(m \times n)$ is a $pm \times qn$ dimensional matrix defined by $A \otimes B = a_{ij}B$, $i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, q$],[18] and the vectors $\boldsymbol{p}$ and $\boldsymbol{q}^*$ are described as

$$\boldsymbol{p} = [P(1,1), P(1,2), P(1,3), \ldots, P(1,n), P(2,2), \ldots, P(n,n)]^T$$

$$\boldsymbol{q}^* = [Q^*(1,1), Q^*(1,2), \ldots, Q^*(n,n)]^T$$

It is well known[19] that there exists a solution of Eq. (A3) if and only if the vector $\boldsymbol{q}^*$ is in the range of $\bar{F}$, denoted by $R(\bar{F})$. Alternatively, the rank of the augmented matrix $[\bar{F}, \boldsymbol{q}^*]$ is equal to the rank of $\bar{F}$. This condition can also be shown[18] to be equivalent to

$$|\lambda_i(F)\lambda_j(F)| \neq 1 \qquad \forall \quad i, j = 1, 2, \ldots, n \tag{A4}$$

where $\lambda_i(F)$ is the ith eigenvalue of the matrix F.

For the solution of Eq. (A3) to be unique, it is necessary that the matrix $\bar{F}$ be nonsingular. Alternatively, it can be shown[20] that such a solution is unique if the eigenvalues of F are such that

$$|\lambda_i(F)\lambda_j(F)| < 1 \qquad \forall \quad i, j = 1, 2, \ldots, n \tag{A5}$$

However, it is sufficient to show that

$$|\lambda_m(F)| < 1 \qquad m = 1, 2, \ldots, n$$

For the solution of Eq. (A2) to be positive definite, it is necessary that the matrix Q^* be positive definite. Thus, the following theorem, whose formal proof is omitted, can be stated[19]:

Theorem 2. Let F and Q^* be $n \times n$ matrices and let Q^* be positive definite. Then 1) if F is stable with respect to the unit circle, Eq. (A2) has a unique solution P, and P is positive definite; conversely, 2) if there is a positive definite matrix P satisfying Eq. (A2), then F is stable with respect to the unit circle, or $|\lambda_i(F)| < 1$.

Algebraic Solution of Discrete Lyapunov Equation

For investigating the stability of a low-order dynamical system, inversion of the matrix $\bar{F}$ required to obtain the solution[21]

$$\boldsymbol{p} = \bar{F}^{-1}\boldsymbol{q}^* \tag{A6}$$

may not be difficult. But for systems of large dimension, the algebra becomes unwieldy unless the matrix $\bar{F}$ is in a special form. To overcome this difficulty, the matrix bilinear transformation[22]

$$A = (F - I)^{-1}(F + I) \tag{A7}$$

is used to transform Eq. (A2) into

$$AP + PA^T = \bar{Q} \tag{A8}$$

where

$$\bar{Q} = -2(F - I)^{-1}Q^*(F - I)^{-1^T} \tag{A9}$$

Solution of this equation was obtained in Ref. 23 while investigating the stability of a second-order continuous time linear dynamical system. The solution was derived by direct matrix inversion and the authors[23] showed that when $\bar{Q} = I$, the solution is positive definite if and only if

$$\operatorname{tr} A > 0 \tag{A10}$$

and

$$\det A > 0 \tag{A11}$$

As mentioned earlier, this approach is unwieldy for higher dimensional systems, and several alternative techniques to solve Eq. (A8) analytically and numerically exist in the literature. An interesting algorithm for solving the equation

$$AP + PB = \bar{Q} \tag{A12}$$

is given in Ref. 24, where P and $\bar{Q}$ are $M \times N$ matrices, A is an $M \times M$ matrix and B is an $N \times N$ matrix. This algorithm is particularly attractive because it involves inversion of either A or B, which are of lower dimension than $\bar{F}$, which in this case is given by $\bar{F} = A \otimes I + I \otimes B^T$.

Specializing this solution to the second-order system, where $B = A^T$, it can be shown that the solution of Eq. (A8) can be reduced to

$$P = (-1/2\operatorname{tr} A)[\bar{Q} + (\det A)A^{-1}\bar{Q}(A^{-1})^T] \tag{A13}$$

It is readily seen that the solution exists if and only if Eqs. (A10) and (A11) are satisfied. Performing the multiplications indicated in Eq. (A13), it is possible to obtain

$$\begin{bmatrix} P(1,1) \\ P(1,2) \\ P(2,1) \\ P(2,2) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} A^2(2,2) + \det A & -A(1,2)A(2,2) & -A(1,2)A(2,2) & A^2(1,2) \\ -A(2,1)A(2,2) & A(1,1)A(2,2) + \det A & A(1,2)A(2,1) & -A(1,1)A(1,2) \\ -A(2,1)A(2,2) & A(1,2)A(2,1) & A(1,1)A(2,2) + \det A & -A(1,1)A(1,2) \\ A^2(2,1) & -A(2,1)A(1,1) & -A(1,1)A(2,1) & A^2(1,1) + \det A \end{bmatrix} \begin{bmatrix} \bar{Q}(1,1) \\ \bar{Q}(1,2) \\ \bar{Q}(2,1) \\ \bar{Q}(2,2) \end{bmatrix} \tag{A14}$$

where $\Delta = 2 \operatorname{tr} A \det A$.

This solution is interesting because it can be written directly when the elements of the matrix A are known and only the trace and determinant of the matrix A need to be computed. Substituting for A and $\bar{Q}$ from Eqs. (A7) and (A9), respectively, the complete solution, where

$$\Delta = (1 - \det F)\det(F - I)\det(F + I)$$

is given by

$$\begin{bmatrix} P^C(1,1) \\ P^C(1,2) \\ P^C(2,1) \\ P^C(2,2) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} [1+F^2(2,2)](1+\det F) - 2F(2,2)\mathrm{tr}F & -F(1,2)[(1+\det F)F(2,2) - \mathrm{tr}F] & -F(1,2)[F(2,2)(1+\det F) - \mathrm{tr}F] & F^2(1,2)(1+\det F) \\ -F(2,1)[F(2,2)(1+\det F) - \mathrm{tr}F] & [1+F(1,1)F(2,2)](1+\det F) - (\mathrm{tr}F)^2 & F(1,2)F(2,1)(1+\det F) & -F(1,2)[F(1,1)(1+\det F) - \mathrm{tr}F] \\ -F(2,1)[F(2,2)(1+\det F) - \mathrm{tr}F] & F(1,2)F(2,1)(1+\det F) & [1+F(1,1)F(2,2)](1+\det F) - (\mathrm{tr}F)^2 & [1+F(1,1)F(2,2)](1+\det F) - (\mathrm{tr}F)^2 \\ F^2(2,1)(1+\det F) & -F(2,1)[F(1,1)(1+\det F) - \mathrm{tr}F] & -F(2,1)[F(1,1)(1+\det F) - \mathrm{tr}F] & [1+F^2(1,1)](1+\det F) - 2F(1,1)\mathrm{tr}F \end{bmatrix} \begin{bmatrix} Q^*(1,1) \\ Q^*(1,2) \\ Q^*(2,1) \\ Q^*(2,2) \end{bmatrix} \tag{A15}$$

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